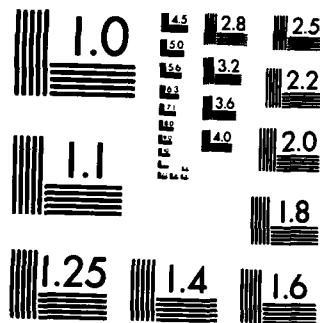


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# Variability of Measures of Weapons Effectiveness

## Volume II: Application to Blast Sensitive Targets in the Absence of Delivery Error

B D Sivazlian

UNIVERSITY OF FLORIDA  
DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING  
GAINESVILLE, FLORIDA 32611

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<p>The problem of computing the uncertainty associated with the probability of kill <math>P_{kb}</math> due to blast in the absence of weapon fragmentation and aiming error is considered. The damage function due to blast is assumed to be a piecewise linear function of the distance between the blast point and the target point. Under the assumption that the input parameters are uniformly distributed, explicit expressions are obtained for the expectation and variance of <math>P_{kb}</math>.</p>				
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Application to Blast Sensitive Targets in the Absence of Delivery Error

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## PREFACE

This report describes work done in the summer of 1983 by Dr B. D. Sivazlian, Department of Industrial and Systems Engineering, the University of Florida, Gainesville, Florida 32611 under Contract No. F08635-83-C-0202 with the Air Force Armament Laboratory (AFATL), Armament Division, Eglin Air Force Base, Florida 32542. The program manager was Mr Daniel A. McInnis (DLYW).

The work was initiated under a 1982 USAF-SCEEE Summer Faculty Research Program sponsored by the Air Force Office of Scientific Research conducted by the Southeastern Center for Electrical Engineering Education (SCEEE) under Contract No. F49620-82-C-0035.

This work addresses itself to the problem of computing the uncertainty associated with the probability of kill  $P_{kb}$  due to blast alone in the absence of weapon fragmentation and aiming error. The assumption is made that (1)  $P_{kb}$  is unity between the center of the blast and a distance  $A$  from the center, (2)  $P_{kb}$  is negligible beyond a certain distance  $B$  from the center ( $B > A$ ), (3)  $P_{kb}$  decreases linearly between  $A$  and  $B$ , and (4)  $A$  and/or  $B$  are random variables uniformly distributed over given ranges. Explicit expressions for  $E[P_{kb}]$  and  $E[P_{kb}^2]$  are derived for various weapon target locations. ( $E[\cdot]$ , as usual, is the expectation operator.)

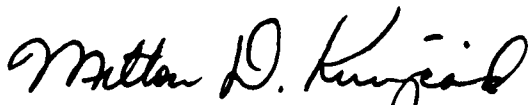
The author has benefited from helpful discussions with several people. Particular thanks are due to Mr Jerry Bass, Mr Daniel McInnis and Mr Charles Reynolds, all from DLYW, who have read the report and have contributed to it through helpful comments.

The report is the second of a series dealing with the uncertainty associated with various weapon effectiveness indices and details methodologies and techniques used in computing such uncertainties in the presence of error in the input parameters.

The Public Affairs Office has reviewed this report, and it is releasable to the National Technical Information Service (NTIS), where it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

A handwritten signature in cursive script, reading "Milton D. Kingcaid".

MILTON D. KINGCAID, Colonel, USAF  
Chief, Analysis and Strategic Defense Division

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## SECTION I

### INTRODUCTION

When for an exploding weapon the effect of fragmentation is negligible, a target may be damaged or destroyed due to the prevailing blast. If the target is close to the location of the weapon, the probability that the target will be destroyed is unity. The effect of the blast prevails highly up to a certain distance from the center of the exploding weapon, and after some point, it decreases. The damage becomes negligible or non-existent beyond a certain distance. It is customary to describe the probability of kill  $P_{kb}$  due to blast as a function of the distance between the center of the weapon and the center of the target by a graph as shown in Figure 1. Analytically,  $P_{kb}$  as a function of distance  $R$  has the form

$$P_{kb} = \begin{cases} 1 & 0 < R < A \\ 1 - \frac{R-A}{B-A} & A < R < B \\ 0 & B < R < \infty \end{cases} \quad (1)$$

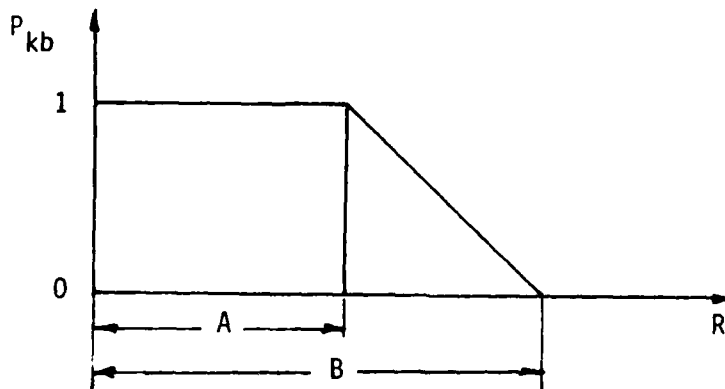


FIGURE 1. Probability of Kill due to Blast as a Function of the Distance Between the Center of the Weapon and the Center of the Target.

A is the maximum distance at which irreparable damage is experienced, and B is the minimum distance at which no damage occurs. It is assumed that  $P_{kb}$  decreases linearly between the distances A and B.

The distances A and B are obtained experimentally for a given weapon/target combination and are subject to measurement error. Thus, A and B are, in general, random variables. The nature of the distribution of A and B will depend on several factors such as the weapon, the target, the atmospheric conditions, etc. In the present work it shall simply be assumed that if A is a random variable, it is uniformly distributed in the interval  $A_1 < A < A_2$ . Similarly, if B is a random variable, it is uniformly distributed in the interval  $B_1 < B < B_2$ . The assumption is further made that the four quantities  $A_1$ ,  $A_2$ ,  $B_1$ , and  $B_2$  can be determined and that  $A_1 < A_2 < B_1 < B_2$ .

Under these assumptions and conditions, the questions to be answered consist in estimating the mean and variance of  $P_{kb}$ , the probability of kill due to blast. It is evident that  $P_{kb}$  is a random variable and that its estimated values will depend on the relative distance R between the center of the target and the center of the weapon. Hence, as may be expected,  $E[P_{kb}]$  and  $E[P_{kb}^2]$  will assume different mathematical forms depending on the distance R.  $E[\cdot]$ , as usual, represents the expectation operator. The variance of  $P_{kb}$ ,  $\text{Var}[P_{kb}]$  can then be obtained from the well known relation

$$\text{Var}[P_{kb}] = E[P_{kb}^2] - \{E[P_{kb}]\}^2$$

The estimates of  $P_{kb}$  will be investigated for the following three situations:

1. A is a random variable and B is fixed.
2. A is fixed and B is a random variable.
3. A and B are random variables.

SECTION II  
COMPUTATION OF  $E[P_{kb}]$  AND  $E[P_{kb}^2]$

1. A is a random variable and B is fixed

The probability density function of A is assumed to be

$$f_A(x) = \begin{cases} 0 & 0 < x < A_1 \\ \frac{1}{A_2 - A_1} & A_1 < x < A_2 \\ 0 & A_2 < x < \infty \end{cases} \quad (2)$$

Further, the distance B is fixed and known. Thus  $A_1$ ,  $A_2$ , and B are input parameters. The center of the target may be located such that its distance R from the center of the blast satisfies any of the following inequalities:

$$0 < R < A_1, A_1 < R < A_2, A_2 < R < B, \text{ and } B < R < \infty.$$

Each of these four cases is considered separately, and expressions for  $E[P_{kb}]$  and  $E[P_{kb}^2]$  are computed.

Case 1:  $0 < R < A_1$

From relation (1) it is evident that for all R - lying between 0 and the minimum value of A,  $P_{kb} = 1$ . It follows that

$$E[P_{kb}] = 1 \quad \text{and} \quad E[P_{kb}^2] = 1.$$

Case 2:  $A_1 < R < A_2$

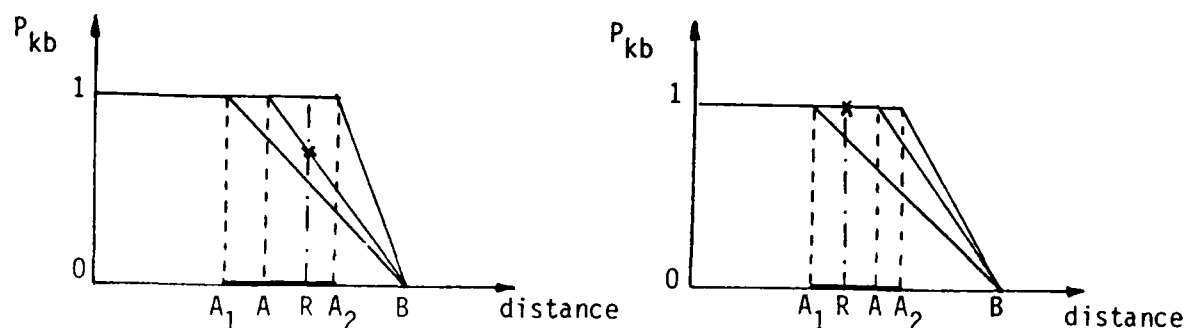


Figure 2.  $P_{kb} = 1 - \frac{R-A}{B-A}$  if  $A_1 < A < R < A_2$

$P_{kb} = 1$  if  $A_1 < R < A < A_2$

From relation (1) and from Figure 2, it is evident that  $P_{kb}$  takes two different values depending on the relative position of  $R$  with respect to  $A_1$ ,  $A$  and  $A_2$ . Hence,

$$P_{kb} = \begin{cases} 1 - \frac{R-A}{B-A} & A_1 < A < R < A_2 \\ 1 & A_1 < R < A < A_2 \end{cases} \quad (3)$$

Then,

$$\begin{aligned} E[P_{kb}] &= \int_{A_1}^{A_2} P_{kb} \cdot f_A(x) dx \\ &= \int_{A_1}^R \left(1 - \frac{R-x}{B-x}\right) \frac{1}{A_2-A_1} dx + \int_R^{A_2} 1 \cdot \frac{1}{A_2-A_1} dx \\ &= \int_{A_1}^R \frac{B-R}{B-x} \cdot \frac{1}{A_2-A_1} dx + \frac{A_2-R}{A_2-A_1} \\ &= \frac{1}{A_2-A_1} \left[ (B-R) \ln \frac{B-A_1}{B-R} + (A_2-R) \right] \end{aligned} \quad (4)$$

Also

$$\begin{aligned}
 E[p_{kb}^2] &= \int_{A_1}^{A_2} p_{kb}^2 \cdot f_A(x) dx \\
 &= \int_{A_1}^R \left(1 - \frac{R-x}{B-x}\right)^2 \frac{1}{A_2-A_1} dx + \int_R^{A_2} 1^2 \cdot \frac{1}{A_2-A_1} dx \\
 &= \frac{(B-R)}{(A_2-A_1)} \int_{A_1}^R \frac{1}{(B-x)^2} dx + \frac{A_2-R}{A_2-A_1} \\
 &= \frac{(B-R)}{(A_2-A_1)} \left[ \frac{1}{B-R} - \frac{1}{B-A_1} \right] + \frac{A_2-R}{A_2-A_1} \\
 &= \frac{1}{A_2-A_1} \left[ \frac{(B-R)(R-A_1)}{(B-A_1)} + (A_2-R) \right] \quad (5)
 \end{aligned}$$

Case 3:  $A_2 < R < B$

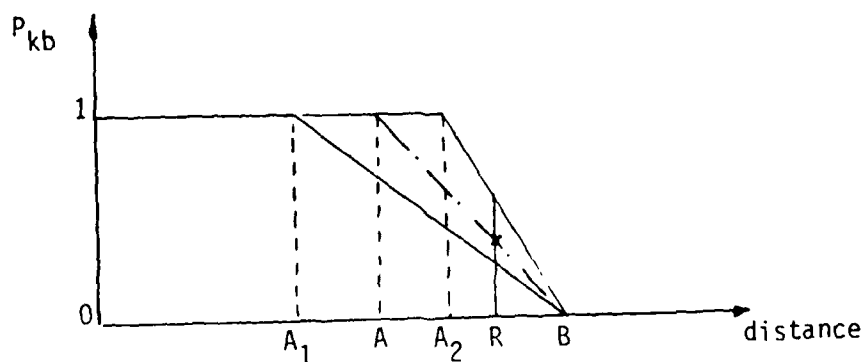


Figure 3.  $p_{kb} = 1 - \frac{R-A}{B-A}$  if  $A_1 < A < A_2 < R < B$

For this particular case, it may be noted from Figure 3 that

$$P_{kb} = 1 - \frac{R-A}{B-A}.$$

Hence,

$$\begin{aligned} E[P_{kb}] &= \int_{A_1}^{A_2} P_{kb} f_A(x) dx \\ &= \int_{A_1}^{A_2} \left(1 - \frac{R-x}{B-x}\right) \cdot \frac{1}{A_2-A_1} dx \\ &= \frac{B-R}{A_2-A_1} \ln \frac{B-A_1}{B-A_2}. \end{aligned} \quad (5)$$

Similarly,

$$\begin{aligned} E[P_{kb}^2] &= \int_{A_1}^{A_2} P_{kb}^2 \cdot f_A(x) dx \\ &= \int_{A_1}^{A_2} \left(1 - \frac{R-x}{B-x}\right)^2 \frac{1}{A_2-A_1} dx \\ &= \frac{(B-R)^2}{A_2-A_1} \int_{A_1}^{A_2} \frac{dx}{(B-x)^2} \\ &= \frac{(B-R)^2}{(A_2-A_1)} \left(\frac{1}{B-A_2} - \frac{1}{B-A_1}\right) \\ &= \frac{(B-R)^2}{(B-A_2)(B-A_1)}. \end{aligned} \quad (7)$$

Case 4:  $B < R < \infty$

Since for this case  $P_{kb} = 0$ , it follows that

$$E[P_{kb}] = 0 \quad \text{and} \quad E[P_{kb}^2] = 0.$$

The results obtained so far for  $E[P_{kb}]$  and  $E[P_{kb}^2]$  are summarized on the following page. Note that at the values of  $R = A_1$ ,  $R = A_2$  and  $R = B$ , both  $E[P_{kb}]$  and  $E[P_{kb}^2]$  are continuous function of  $R$ .

$$E[P_{kb}] = \begin{cases} 1 & 0 < R < A_1 \\ \frac{1}{(A_2 - A_1)} \left[ (B - R) \ln \frac{B - A_1}{B - R} + (A_2 - R) \right] & A_1 < R < A_2 \\ \frac{B - R}{A_2 - A_1} \ln \frac{B - A_1}{B - R} & A_2 < R < B \\ 0 & B < R < \infty \end{cases} \quad (8)$$

$$E[P_{kb}^2] = \begin{cases} 1 & 0 < R < A_1 \\ \frac{1}{A_2 - A_1} \left[ \frac{(B - R)(R - A_1)}{(B - A_1)} + (A_2 - R) \right] & A_1 < R < A_2 \\ \frac{(B - R)^2}{(B - A_2)(B - A_1)} & A_2 < R < B \\ 0 & B < R < \infty \end{cases} \quad (9)$$

## 2. A is fixed and B is a random variable

The probability density function of B is assumed to be

$$f_B(y) = \begin{cases} 0 & 0 < y < B_1 \\ \frac{1}{B_2 - B_1} & B_1 < y < B_2 \\ 0 & B_2 < y < \infty \end{cases} \quad (10)$$



Further, the distance  $A$  is fixed and known. Thus,  $A$ ,  $B_1$ , and  $B_2$  are input parameters. The center of the target may be located such that its distance  $R$  from the center of the blast satisfies any of the following inequalities:

$$0 < R < A, \quad A < R < B_1, \quad B_1 < R < B_2, \quad \text{and} \quad B_2 < R < \infty.$$

Each of these four cases is considered separately, and expressions for  $E[P_{kb}]$  and  $E[P_{kb}^2]$  are computed.

Case 1:  $0 < R < A$

From relation (1), it is evident that for all  $R$  lying between 0 and the distance  $A$ ,  $P_{kb} = 1$ . Hence,

$$E[P_{kb}] = 1 \quad \text{and} \quad E[P_{kb}^2] = 1.$$

Case 2:  $A < R < B_1$

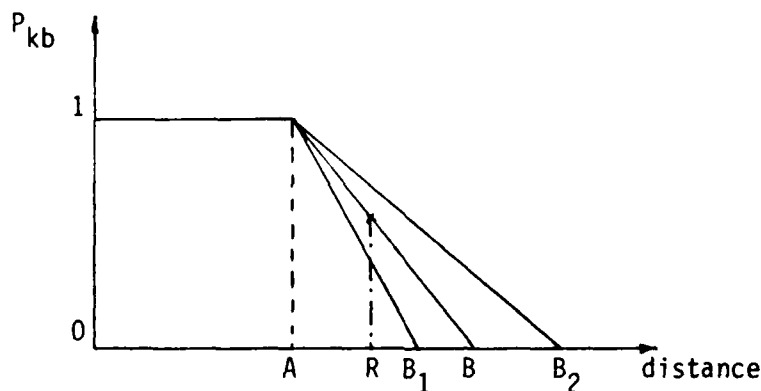


Figure 4.  $P_{kb} = 1 - \frac{R-A}{B-A}$  if  $A < R < B_1$

It may be noted from the figure that in this case

$$P_{kb} = 1 - \frac{R-A}{B-A}.$$

Hence,

$$\begin{aligned}
 E[P_{kb}] &= \int_{B_1}^{B_2} P_{kb} \cdot f_B(y) dy \\
 &= \int_{B_1}^{B_2} \left(1 - \frac{R-A}{y-A}\right) \frac{1}{B_2-B_1} dy \\
 &= 1 - \frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{B_1-A}.
 \end{aligned} \tag{11}$$

Similarly,

$$\begin{aligned}
 E[P_{kb}^2] &= \int_{B_1}^{B_2} P_{kb}^2 f_B(y) dy \\
 &= \int_{B_1}^{B_2} \left(1 - \frac{R-A}{y-A}\right)^2 \frac{1}{B_2-B_1} dy \\
 &= \int_{B_1}^{B_2} \left[1 - 2 \frac{R-A}{y-A} + \frac{(R-A)^2}{(y-A)^2}\right] \frac{1}{B_2-B_1} dy \\
 &= 1 - 2 \frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{B_1-A} + \frac{(R-A)^2}{(B_2-B_1)} \left[-\frac{1}{B_2-A} + \frac{1}{B_1-A}\right] \\
 &= 1 - 2 \frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{B_1-A} + \frac{(R-A)^2}{(B_2-A)(B_1-A)}.
 \end{aligned} \tag{12}$$

Case 3:  $B_1 < R < B_2$

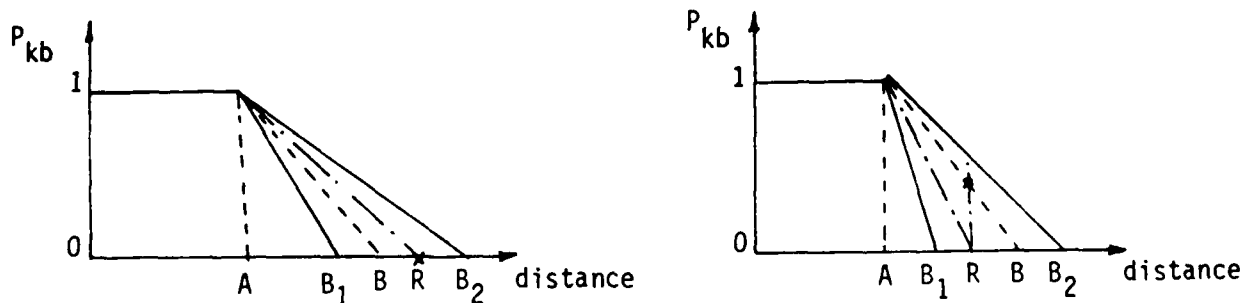


Figure 5.  $P_{kb} = 0$  if  $B_1 < B < R < B_2$   
 $P_{kb} = 1 - \frac{R-A}{B-A}$  if  $B_1 < R < B < B_2$

From relation (1) and from Figure 5, it is evident that  $P_{kb}$  takes on two different values depending on the relative position of  $R$  with respect to  $B_1$ ,  $B$ , and  $B_2$ . Hence

$$P_{kb} = \begin{cases} 0 & B_1 < B < R < B_2 \\ 1 - \frac{R-A}{B-A} & B_1 < R < B < B_2 \end{cases} \quad (13)$$

Then,

$$\begin{aligned} E[P_{kb}] &= \int_{B_1}^{B_2} P_{kb} \cdot f_B(y) dy \\ &= \int_{B_1}^R 0 \cdot \frac{1}{B_2-B_1} dy + \int_R^{B_2} \left(1 - \frac{R-A}{y-A}\right) \frac{1}{B_2-B_1} dy \\ &= \frac{B_2-R}{B_2-B_1} - \frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{R-A} \end{aligned} \quad (14)$$

Similarly,

$$\begin{aligned} E[P_{kb}^2] &= \int_{B_1}^{B_2} P_{kb}^2 f_B(y) dy \\ &= \int_{B_1}^R 0^2 \cdot \frac{1}{B_2-B_1} dy + \int_R^{B_2} \left(1 - \frac{R-A}{y-A}\right)^2 \frac{1}{B_2-B_1} dy \\ E[P_{kb}^2] &= \int_R^{B_2} \left[1 - 2 \frac{R-A}{y-A} + \frac{(R-A)^2}{(y-A)^2}\right] \frac{1}{B_2-B_1} dy \\ &= \frac{1}{B_2-B_1} \left[ (B_2-R) - 2(R-A) \ln \frac{B_2-A}{R-A} - (R-A)^2 \left( \frac{1}{B_2-A} - \frac{1}{R-A} \right) \right] \\ &= \frac{1}{B_2-B_1} \left[ (B_2-R) - 2(R-A) \ln \frac{B_2-A}{R-A} + \frac{(R-A)(B_2-R)}{(B_2-A)} \right] \end{aligned} \quad (15)$$

Case 4:  $B_2 < R < \infty$

Clearly  $P_{kb} = 0$ , hence

$$E[P_{kb}] = 0 \quad \text{and} \quad E[P_{kb}^2] = 0.$$

The results obtained so far for  $E[P_{kb}]$  and  $E[P_{kb}^2]$  are summarized on the following page. Note that at the values of  $R = A$ ,  $R = B_1$ , and  $R = B_2$ , both  $E[P_{kb}]$  and  $E[P_{kb}^2]$  remain continuous functions of the variable  $R$ .

$$E[P_{kb}] = \begin{cases} 1 & 0 < R < A \\ 1 - \frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{B_1-A} & A < R < B_1 \\ \frac{B_2-R}{B_2-B_1} - \frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{R-A} & B_1 < R < B_2 \\ 0 & B_2 < R < \infty \end{cases} \quad (16)$$

$$E[P_{kb}^2] = \begin{cases} 1 & 0 < R < A \\ 1 - 2 \frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{B_1-A} + \frac{(R-A)^2}{(B_2-A)(B_1-A)} & A < R < B_1 \\ \frac{B_2-R}{B_2-B_1} - 2 \frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{R-A} + \frac{(R-A)(B_2-R)}{(B_2-B_1)(B_2-A)} & B_1 < R < B_2 \\ 0 & B_2 < R < \infty \end{cases} \quad (17)$$

### 3. A and B are random variables

It will be assumed that the probability density functions of  $A$  and  $B$  are given, respectively, by (2) and (10). The input parameters to the problem are  $A_1$ ,  $A_2$ ,  $B_1$ , and  $B_2$  where  $A_1 < A_2 < B_1 < B_2$ . Again, here the center of the target may be located such that its distance  $R$  from the center of the blast satisfies any of the following inequalities:

$$0 < R < A_1, \quad A_1 < R < A_2, \quad A_2 < R < B_1, \quad B_1 < R < B_2, \quad B_2 < R < \infty.$$

Each of these five cases is considered separately, and expressions for  $E[P_{kb}]$  and  $E[P_{kb}^2]$  are computed.

Case 1:  $0 < R < A_1$

Since for this case  $P_{kb} = 1$ , it follows that

$$E[P_{kb}] = 1 \quad \text{and} \quad E[P_{kb}^2] = 1.$$

Case 2:  $A_1 < R < A_2$

It is evident that

$$P_{kb} = \begin{cases} 1 - \frac{R-A}{B-A} & \text{if } A_1 < A < R < A_2 \\ 1 & \text{if } A_1 < R < A < A_2. \end{cases}$$

Hence, from (4)

$$\begin{aligned} E[P_{kb}] &= \int_{B_1}^{B_2} \int_{A_1}^{A_2} P_{kb} \cdot \frac{1}{A_1-A_2} \cdot \frac{1}{B_1-B_2} dx dy \\ &= \int_{B_1}^{B_2} \frac{1}{B_2-B_1} \left\{ \int_{A_1}^R \left(1 - \frac{R-x}{y-x}\right) \frac{1}{A_2-A_1} dx + \int_R^{A_2} 1 \cdot \frac{1}{A_2-A_1} dx \right\} dy \\ &= \frac{1}{(A_2-A_1)(B_2-B_1)} \int_{B_1}^{B_2} \left[ (y-R) \ln \frac{y-A_1}{y-R} + (A_2-R) \right] dy \\ &= \frac{1}{(A_2-A_1)(B_2-B_1)} \left\{ \int_{B_1}^{B_2} (y-R) \ln (y-A_1) dy \right. \\ &\quad \left. - \int_{B_1}^{B_2} (y-R) \ln (y-R) dy \right\} + \frac{A_2-R}{A_2-A_1}. \end{aligned} \tag{18}$$

Each of the integrals on the right-hand side of (18) is considered separately.

For the first integral one obtains, after integrating by parts,

$$\begin{aligned}
& \int_{B_1}^{B_2} (y-R) \ln (y-A_1) dy = \frac{(y-R)^2}{2} \ln (y-A_1) \Big|_{B_1}^{B_2} \\
& - \int_{B_1}^{B_2} \frac{(y-R)}{2(y-A_1)} dy \\
& = \frac{(B_2-R)^2}{2} \ln (B_2-A_1) - \frac{(B_1-R)^2}{2} \ln (B_1-A_1) \\
& - \frac{1}{2} \int_{B_1}^{B_2} \left[ y + A_1 - 2R + \frac{(R-A_1)^2}{(y-A_1)} \right] dy \\
& = \frac{(B_2-R)^2}{2} \ln (B_2-A_1) - \frac{(B_1-R)^2}{2} \ln (B_1-A_1) \\
& - \frac{1}{2} \left[ \frac{y^2}{2} + (A_1-2R)y + (R-A_1)^2 \ln (y-A_1) \right]_{B_1}^{B_2} \\
& = \frac{(B_2-R)^2}{2} \ln (B_2-A_1) - \frac{(B_1-R)^2}{2} \ln (B_1-A_1) \\
& - \frac{1}{4} (B_2^2 - B_1^2) - \frac{1}{2} (A_1-2R) (B_2-B_1) - \frac{1}{2} (R-A_1)^2 \ln \frac{B_2-A_1}{B_1-A_1} . \quad (19)
\end{aligned}$$

The value of the second integral in (18) can be obtained from (19) by substituting  $R$  to  $A_1$ . This yields

$$\begin{aligned}
& \int_{B_1}^{B_2} (y-R) \ln (y-R) dy = \\
& \frac{(B_2-R)^2}{2} \ln (B_2-R) - \frac{(B_1-R)^2}{2} \ln (B_1-R) - \frac{B_2^2 - B_1^2}{4} + \frac{R(B_2-B_1)}{2} . \quad (20)
\end{aligned}$$

Substituting (19) and (20) in (18) results in:

$$\begin{aligned}
E[P_{kb}] = & \frac{1}{(A_2-A_1)(B_2-B_1)} \left\{ \frac{(B_2-R)^2}{2} \ln(B_2-A_1) - \frac{(B_1-R)^2}{2} \ln(B_1-A_1) \right. \\
& - \frac{1}{4} (B_2^2-B_1^2) - \frac{1}{2} (A_1-2R)(B_2-B_1) - \frac{1}{2} (R-A_1)^2 \ln \frac{B_2-A_1}{B_1-A_1} \\
& - \left[ \frac{(B_2-R)^2}{2} \ln(B_2-R) - \frac{(B_1-R)^2}{2} \ln(B_1-R) - \frac{B_2^2-B_1^2}{4} \right. \\
& \left. \left. + \frac{R(B_2-B_1)}{2} \right] + (A_2-R)(B_2-B_1) \right\}
\end{aligned}$$

which reduces to

$$\begin{aligned}
E[P_{kb}] = & \frac{1}{(A_2-A_1)(B_2-B_1)} \left[ \frac{(B_2-R)^2}{2} \ln \frac{B_2-A_1}{B_2-R} - \frac{(B_1-R)^2}{2} \ln \frac{B_1-A_1}{B_1-R} \right. \\
& \left. - \frac{1}{2} (R-A_1)^2 \ln \frac{B_2-A_1}{B_1-A_1} + \frac{1}{2} (B_2-B_1)(2A_2-A_1-R) \right]. \quad (21)
\end{aligned}$$

To compute  $E[P_{kb}^2]$  expression (5) is referred to and one obtains

$$\begin{aligned}
E[P_{kb}^2] = & \int_{B_1}^{B_2} \int_{A_1}^{A_2} P_{kb}^2 \cdot \frac{1}{A_2-A_1} \cdot \frac{1}{B_2-B_1} dx dy \\
= & \int_{B_1}^{B_2} \left[ \int_{A_1}^R \left(1 - \frac{R-x}{B-x}\right)^2 \frac{1}{A_2-A_1} dx + \int_R^{A_2} 1^2 \cdot \frac{1}{A_2-A_1} dx \right] \frac{1}{B_2-B_1} dy \\
= & \int_{B_1}^{B_2} \left[ \frac{(y-R)(R-A_1)}{(A_2-A_1)(y-A_1)} + \frac{A_2-R}{A_2-A_1} \right] \frac{1}{B_2-B_1} dy \\
= & \frac{(R-A_1)}{(A_2-A_1)(B_2-B_1)} \int_{B_1}^{B_2} \frac{y-R}{y-A_1} dy + \frac{A_2-R}{A_2-A_1} \\
= & \frac{(R-A_1)}{(A_2-A_1)(B_2-B_1)} \int_{B_1}^{B_2} \left[ 1 + \frac{A_1-R}{y-A_1} \right] dy + \frac{A_2-R}{A_2-A_1} \\
E[P_{kb}^2] = & \frac{(R-A_1)}{(A_2-A_1)(B_2-B_1)} \left[ (B_2-B_1) - (R-A_1) \ln(y-A_1) \right] \Big|_{B_1}^{B_2} + \frac{A_2-R}{A_2-A_1} \\
= & 1 - \frac{(R-A_1)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_1}{B_1-A_1}. \quad (22)
\end{aligned}$$

Case 3:  $A_2 < R < B_1$

For this case,  $P_{kb} = 1 - \frac{R-A}{B-A}$ . Thus, using (6) one gets

$$\begin{aligned} E[P_{kb}] &= \int_{B_1}^{B_2} \int_{A_1}^{A_2} \left[1 - \frac{R-x}{y-x}\right] \cdot \frac{1}{A_2-A_1} \cdot \frac{1}{B_2-B_1} dx dy \\ &= \frac{1}{(A_2-A_1)(B_2-B_1)} \int_{B_1}^{B_2} (y-R) \ln \frac{y-A_1}{y-A_2} dy \\ &= \frac{1}{(A_2-A_1)(B_2-B_1)} \left[ \int_{B_1}^{B_2} (y-R) \ln (y-A_1) dy - \int_{B_1}^{B_2} (y-R) \ln (y-A_2) dy \right]. \end{aligned}$$

Using (19) one immediately obtains

$$\begin{aligned} E[P_{kb}] &= \frac{1}{(A_2-A_1)(B_2-B_1)} \left\{ \left[ \frac{(B_2-R)^2}{2} \ln (B_2-A_1) - \frac{(B_1-R)^2}{2} \ln (B_1-A_1) \right. \right. \\ &\quad - \frac{1}{4} (B_2^2-B_1^2) - \frac{1}{2} (A_1-2R)(B_2-B_1) - \frac{1}{2} (R-A_1)^2 \ln \frac{B_2-A_1}{B_1-A_1} \\ &\quad - \left[ \frac{(B_2-R)^2}{2} \ln (B_2-A_2) - \frac{(B_1-R)^2}{2} \ln (B_1-A_2) - \frac{1}{4} (B_2^2-B_1^2) \right. \\ &\quad \left. \left. - \frac{1}{2} (A_2-2R)(B_2-B_1) - \frac{1}{2} (R-A_2)^2 \ln \frac{B_2-A_2}{B_1-A_2} \right] \right\} \\ &= \frac{1}{(A_2-A_1)(B_2-B_1)} \left[ \frac{(B_2-R)^2}{2} \ln \frac{B_2-A_1}{B_2-A_2} - \frac{(B_1-R)^2}{2} \ln \frac{B_1-A_1}{B_1-A_2} \right. \\ &\quad \left. + \frac{1}{2} (B_2-B_1)(A_2-A_1) - \frac{1}{2} (R-A_1)^2 \ln \frac{B_2-A_1}{B_1-A_1} + \frac{(R-A_2)^2}{2} \ln \frac{B_2-A_2}{B_1-A_2} \right]. \quad (23) \end{aligned}$$

The expression for  $E[P_{kb}^2]$  is

$$E[P_{kb}^2] = \int_{B_1}^{B_2} \int_{A_1}^{A_2} \left[1 - \frac{R-x}{y-x}\right] \frac{1}{A_2-A_1} \cdot \frac{1}{B_1-B_2} dx dy.$$



Using (7) results in

$$E[P_{kb}^2] = \frac{1}{B_2 - B_1} \int_{B_1}^{B_2} \frac{(y-R)^2}{(y-A_2)(y-A_1)} dy \quad (24)$$

The integrand in (24) is reduced into partial fraction as follows

$$\begin{aligned} \frac{(y-R)^2}{(y-A_2)(y-A_1)} &= \frac{y^2 - 2Ry + R^2}{(y-A_2)(y-A_1)} = 1 + 1 \\ &= 1 + \frac{(A_1 + A_2 - 2R)y - A_1 A_2 + R^2}{(y-A_2)(y-A_1)} \\ &= 1 + \frac{E}{y-A_2} + \frac{F}{y-A_1} \end{aligned}$$

where E and F are constants whose values are

$$\begin{aligned} E &= \frac{(A_1 + A_2 - 2R) A_2 - A_1 A_2 + R^2}{A_2 - A_1} = \frac{(R - A_2)^2}{A_2 - A_1} \\ F &= \frac{(A_1 + A_2 - 2R) A_1 - A_1 A_2 + R^2}{A_1 - A_2} = \frac{(R - A_1)^2}{A_1 - A_2} \end{aligned}$$

So that

$$\frac{(y-R)^2}{(y-A_2)(y-A_1)} = 1 + \frac{(R-A_2)^2}{A_2-A_1} \cdot \frac{1}{y-A_2} - \frac{(R-A_1)^2}{A_2-A_1} \cdot \frac{1}{y-A_1}.$$

The expression in (24) reduces to

$$\begin{aligned} E[P_{kb}^2] &= \frac{1}{B_2 - B_1} \int_{B_1}^{B_2} \left[ 1 + \frac{(R-A_2)^2}{A_2-A_1} \cdot \frac{1}{y-A_2} - \frac{(R-A_1)^2}{A_2-A_1} \cdot \frac{1}{y-A_1} \right] dy \\ &= 1 + \frac{(R-A_2)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_2}{B_1-A_2} - \frac{(R-A_1)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_1}{B_1-A_1}. \quad (25) \end{aligned}$$

Case 4:  $B_1 < R < B_2$

For this particular situation

$$P_{kb} = \begin{cases} 0 & \text{if } B_1 < B < R < B_2 \\ 1 - \frac{R-A}{B-A} & \text{if } B_1 < R < B < B_2 . \end{cases}$$

The expression for  $E[P_{kb}]$  is

$$E[P_{kb}] = \int_{A_1}^{A_2} \left\{ \int_{B_1}^{B_2} 0 \cdot \frac{1}{B_2-B_1} dy + \int_R^{B_2} \left(1 - \frac{R-x}{y-x}\right) \cdot \frac{1}{B_2-B_1} dy \right\} \frac{1}{A_2-A_1} dx .$$

Using expression (14) one immediately obtains

$$\begin{aligned} E[P_{kb}] &= \int_{A_1}^{A_2} \left[ \frac{B_2-R}{B_2-B_1} - \frac{R-x}{B_2-B_1} \ln \frac{B_2-x}{R-x} \right] \frac{1}{A_2-A_1} dx \\ &= \frac{B_2-R}{B_2-B_1} - \frac{1}{(A_2-A_1)(B_2-B_1)} \left[ \int_{A_1}^{A_2} (R-x) \ln (B_2-x) dx \right. \\ &\quad \left. - \int_{A_1}^{A_2} (R-x) \ln (R-x) dx \right] . \end{aligned} \quad (26)$$

Consider the first integral in (26) and use integration by parts to obtain

$$\begin{aligned} \int_{A_1}^{A_2} (R-x) \ln (B_2-x) dx &= - \frac{(R-x)^2}{2} \ln (B_2-x) \Big|_{A_1}^{A_2} - \int_{A_1}^{A_2} \frac{(R-x)^2}{2(B_2-x)} dx \\ &= - \frac{(R-A_2)^2}{2} \ln (B_2-A_2) + \frac{(R-A_1)^2}{2} \ln (B_2-A_1) \\ &\quad + \frac{1}{2} \int_{A_1}^{A_2} \left[ x + B_2 - 2R - \frac{(B_2-R)^2}{(B_2-x)} \right] dx \end{aligned}$$

$$\begin{aligned}
&= \frac{(R-A_2)^2}{2} \ln (B_2-A_2) + \frac{(R-A_1)^2}{2} \ln (B_2-A_1) \\
&+ \frac{1}{2} \left[ \frac{x^2}{2} + (B_2-2R)x + (B_2-R)^2 \ln (B_2-x) \right] \Big|_{A_1}^{A_2} \\
&= -\frac{(R-A_2)^2}{2} \ln (B_2-A_2) + \frac{(R-A_1)^2}{2} \ln (B_2-A_1) \\
&+ \frac{1}{4} (A_2^2 - A_1^2) + \frac{1}{2} (B_2-2R)(A_2-A_1) + \frac{1}{2} (B_2-R)^2 \ln \frac{B_2-A_2}{B_2-A_1}. \quad (27)
\end{aligned}$$

Consider now the second integral in (26). The value of this integral can be obtained from (27) by substituting  $R$  for  $B_2$ . Thus,

$$\begin{aligned}
\int_{A_1}^{A_2} (R-x) \ln (R-x) dx &= -\frac{(R-A_2)^2}{2} \ln (R-A_2) + \frac{(R-A_1)^2}{2} \ln (R-A_1) \\
&+ \frac{1}{4} (A_2^2 - A_1^2) - \frac{1}{2} R (A_2-A_1). \quad (28)
\end{aligned}$$

The expression for  $E[P_{kb}]$  in (26) becomes after using the results in (27) and (28)

$$\begin{aligned}
E[P_{kb}] &= \frac{1}{(A_2-A_1)(B_2-B_1)} \left\{ (B_2-R)(A_2-A_1) - \left[ -\frac{(R-A_2)^2}{2} \ln (B_2-A_2) \right. \right. \\
&+ \frac{(R-A_1)^2}{2} \ln (B_2-A_1) + \frac{1}{4} (A_2^2 - A_1^2) + \frac{1}{2} (B_2-2R)(A_2-A_1) \\
&+ \frac{1}{2} (B_2-R)^2 \ln \frac{B_2-A_2}{B_2-A_1} \Big] - \frac{(R-A_2)^2}{2} \ln (R-A_2) \\
&+ \frac{(R-A_1)^2}{2} \ln (R-A_1) + \frac{1}{4} (A_2^2 - A_1^2) - \frac{1}{2} R (A_2-A_1) \Big\} \\
&= \frac{1}{(A_2-A_1)(B_2-B_1)} \left[ \frac{1}{2} (A_2-A_1)(B_2-R) + \frac{(R-A_2)^2}{2} \ln \frac{B_2-A_2}{R-A_2} \right. \\
&- \left. \frac{(R-A_1)^2}{2} \ln \frac{B_2-A_1}{R-A_1} - \frac{1}{2} (B_2-R)^2 \ln \frac{B_2-A_2}{B_2-A_1} \right]. \quad (29)
\end{aligned}$$

The expression for  $E[P_{kb}^2]$  is

$$E[P_{kb}^2] = \int_{A_1}^2 \left\{ \int_{B_1}^R 0 \cdot \frac{1}{B_2-B_1} dy + \int_R^{B_2} \left(1 - \frac{R-x}{y-x}\right)^2 \cdot \frac{1}{B_2-B_1} dy \right\} \frac{1}{A_2-A_1} dx .$$

Using expression (15) one obtains

$$\begin{aligned} E[P_{kb}^2] &= \frac{1}{A_2-A_1} \int_{A_1}^2 \frac{1}{B_2-B_1} \left[ (B_2-R) - 2(R-x) \ln \frac{B_2-x}{R-x} + \frac{(R-x)(B_2-R)}{(B_2-x)} \right] dx \\ &= \frac{1}{A_2-A_1} \left\{ \frac{(B_2-R)(A_2-A_1)}{B_2-B_1} - \frac{2}{B_2-B_1} \left[ \int_{A_1}^2 (R-x) \ln (B_2-x) dx \right. \right. \\ &\quad \left. \left. - 2 \int_{A_1}^2 (R-x) \ln (R-x) dx \right] + \frac{B_2-R}{B_2-B_1} \int_{A_1}^2 \frac{R-x}{B_2-x} dx \right\} . \end{aligned}$$

Using (27) and (28) one obtains

$$\begin{aligned} E[P_{kb}^2] &= \frac{1}{A_2-A_1} \left\{ \frac{(B_2-R)(A_2-A_1)}{B_2-B_1} - \frac{2}{B_2-B_1} \left[ -\frac{(R-A_2)^2}{2} \ln (B_2-A_2) \right. \right. \\ &\quad \left. \left. + \frac{(R-A_1)^2}{2} \ln (B_2-A_1) + \frac{1}{4} (A_2^2-A_1^2) + \frac{1}{2} (B_2-2R)(A_2-A_1) \right. \right. \\ &\quad \left. \left. + \frac{1}{2} (B_2-R)^2 \ln \frac{B_2-A_2}{B_2-A_1} \right] + \frac{2}{B_2-B_1} \left[ -\frac{(R-A_2)^2}{2} \ln (R-A_2) \right. \right. \\ &\quad \left. \left. + \frac{(R-A_1)^2}{2} \ln (R-A_1) + \frac{1}{4} (A_2^2-A_1^2) - \frac{1}{2} R(A_2-A_1) \right] \right. \\ &\quad \left. + \frac{B_2-R}{B_2-B_1} \left[ (A_2-A_1) + (R-B_2) \ln \frac{B_2-A_1}{B_2-A_2} \right] \right\} . \end{aligned}$$

Upon simplification, the following result is obtained

$$\begin{aligned} E[P_{kb}^2] &= \frac{B_2-R}{B_2-B_1} + \frac{1}{(A_2-A_1)(B_2-B_1)} \left[ (R-A_2)^2 \ln \frac{B_2-A_2}{R-A_2} \right. \\ &\quad \left. - (R-A_1)^2 \ln \frac{B_2-A_1}{R-A_1} \right] . \end{aligned} \quad (30)$$

Case 5:  $B_2 < R < \infty$

Here  $P_{kb} = 0$  and it follows that

$$E[P_{kb}] = 0 \quad \text{and} \quad E[P_{kb}^2] = 0.$$

The results obtained so far for  $E[P_{kb}]$  and  $E[P_{kb}^2]$  when A and B are random variables are summarized on the following pages. Note that both  $E[P_{kb}]$  and  $E[P_{kb}^2]$  are a continuous function of R.

$$E[P_{kb}] = \begin{cases} (1) & 1 & 0 < R < A_1 \\ (2) & \frac{1}{(A_2 - A_1)(B_2 - B_1)} \left[ \frac{(B_2 - R)^2}{2} \ln \frac{B_2 - A_1}{B_2 - R} - \frac{(B_1 - R)^2}{2} \ln \frac{B_1 - A_1}{B_1 - R} \right. \\ & \left. - \frac{1}{2} (R - A_1)^2 \ln \frac{B_2 - A_1}{B_1 - A_1} + \frac{1}{2} (B_2 - B_1)(2A_2 - A_1 - R) \right] & A_1 < R < A_2 \\ (3) & \frac{1}{(A_2 - A_1)(B_2 - B_1)} \left[ \frac{(B_2 - R)^2}{2} \ln \frac{B_2 - A_1}{B_2 - A_2} - \frac{(B_1 - R)^2}{2} \ln \frac{B_1 - A_1}{B_1 - A_2} \right. \\ & \left. + \frac{1}{2} (B_2 - B_1)(A_2 - A_1) - \frac{1}{2} (R - A_1)^2 \ln \frac{B_2 - A_1}{B_1 - A_1} \right. \\ & \left. + \frac{1}{2} (R - A_2)^2 \ln \frac{B_2 - A_2}{B_1 - A_2} \right] & A_2 < R < B_1 \\ (4) & \frac{1}{(A_2 - A_1)(B_2 - B_1)} \left[ \frac{(R - A_2)^2}{2} \ln \frac{B_2 - A_2}{R - A_2} - \frac{(R - A_1)^2}{2} \ln \frac{B_2 - A_1}{R - A_1} \right. \\ & \left. - \frac{1}{2} (B_2 - R)^2 \ln \frac{B_2 - A_2}{B_2 - A_1} + \frac{1}{2} (A_2 - A_1)(B_2 - R) \right] & B_1 < R < B_2 \\ (5) & 0 & B_2 < R < \infty \end{cases} \quad (31)$$

$$E[P_{kb}^2] = \left\{ \begin{array}{ll} (1) 1 & 0 < R < A_1 \\ (2) 1 - \frac{(R-A_1)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_1}{B_1-A_1} & A_1 < R < A_2 \\ (3) 1 - \frac{(R-A_1)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_1}{B_1-A_1} \\ \quad + \frac{(R-A_2)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_2}{B_1-A_2} & A_2 < R < B_1 \\ (4) \frac{B_2-R}{B_2-B_1} - \frac{(R-A_1)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_1}{R-A_1} \\ \quad + \frac{(R-A_2)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_2}{R-A_2} & B_1 < R < B_2 \\ (5) 0 & B_2 < R < \infty \end{array} \right.$$

(32)

#### 4. Example

Assume that an explosive charge detonates and that it is required to determine the impact of the resulting blast on a target located 19 ft away from the center of the blast. The effect of fragmentation is neglected. It is estimated that if the target is located at a distance of 15 ft  $\pm$  2 ft, the damage due to blast is irreparable. While if the target is located at a distance of 21 ft  $\pm$  1 ft, the damage is negligible. It is required to estimate the probability that the target will be damaged.

Assume that A is uniformly distributed in the interval 13 < A < 17, and that B is uniformly distributed in the interval 20 < B < 22. Clearly A<sub>1</sub>=13 ft, A<sub>2</sub>=17 ft, B<sub>1</sub>=20 ft, B<sub>2</sub>=22 ft, and R=19 ft. Note that

$A_2 < R < B_1$ . This corresponds to the case when A and B are random variables and  $A_2 < R < B_1$ . Formulas (31)-3 and (32)-3 are pertinent and one obtains for  $E[P_{kb}]$  and  $E[P_{kb}^2]$  the following

$$\begin{aligned}
 E[P_{kb}] &= \frac{1}{(A_2-A_1)(B_2-B_1)} \left[ \frac{(B_2-R)^2}{2} \ln \frac{B_2-A_1}{B_2-A_2} \right. \\
 &\quad - \frac{(B_1-R)^2}{2} \ln \frac{B_1-A_1}{B_1-A_2} + \frac{1}{2} (B_2-B_1)(A_2-A_1) \\
 &\quad - \frac{1}{2} (R-A_1)^2 \ln \frac{B_2-A_1}{B_1-A_1} + \frac{1}{2} (R-A_2)^2 \ln \frac{B_2-A_2}{B_1-A_2} \left. \right] \\
 &= \frac{1}{(17-13)(22-20)} \left[ \frac{(22-19)^2}{2} \ln \frac{22-13}{22-17} \right. \\
 &\quad - \frac{(20-19)^2}{2} \ln \frac{20-13}{20-17} + \frac{1}{2} (22-20)(17-13) \\
 &\quad - \frac{1}{2} (19-13)^2 \ln \frac{22-13}{20-13} + \frac{1}{2} (19-17) \ln \frac{22-17}{20-17} \left. \right] \\
 &= \frac{1}{8} \left[ \frac{9}{2} \ln \frac{9}{5} - \frac{1}{2} \ln \frac{7}{3} + 4 - \frac{36}{2} \ln \frac{9}{7} + \frac{4}{2} \ln \frac{5}{3} \right] \\
 E[P_{kb}] &= \frac{1}{8} [(4.5)(.5878) - \frac{1}{2} (.8473) + 4 - 18(.2513) + 2(.5108)] \\
 &= \frac{1}{8} (2.6451 - .4237 + 4 - 4.5234 + 1.0216) \\
 &= .33995
 \end{aligned}$$

$$\begin{aligned}
 E[P_{kb}^2] &= 1 - \frac{(R-A_1)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_1}{B_1-A_1} \\
 &\quad + \frac{(R-A_2)^2}{(A_2-A_1)(B_2-B_1)} \ln \frac{B_2-A_2}{B_1-A_2}
 \end{aligned}$$

$$\begin{aligned}
&= 1 - \frac{(19-13)^2}{(17-13)(22-20)} \ln \frac{22-13}{20-13} \\
&+ \frac{(19-17)^2}{(17-13)(22-20)} \ln \frac{22-17}{20-17} \\
&= 1 - \frac{36}{8} \ln \frac{9}{7} + \frac{4}{8} \ln \frac{5}{3} \\
&= 1 - (4.5) (.2513144) + (.5) (.5108256) \\
&= .124498 .
\end{aligned}$$

To obtain the variance of  $P_{kb}$ , the following formula is used

$$\begin{aligned}
\text{Var}[P_{kb}] &= E[P_{kb}^2] - \{E[P_{kb}]\}^2 \\
&= .124498 - (.33995)^2 \\
&= .008932
\end{aligned}$$

$$\text{Let } \sigma_{P_{kb}} = \sqrt{\text{Var}[P_{kb}]} = .09451 .$$

Then for a two-standard deviation confidence interval one has

$$\begin{aligned}
P_{kb} &= E[P_{kb}] \pm 2 \sigma_{P_{kb}} \\
&= .33995 \pm (2)(.09451) \\
&= .33995 \pm .18902.
\end{aligned}$$

According to Chebyshev's inequality, the probability is at least 75 percent that  $P_{kb}$  will lie in the above interval.



### SECTION III

#### CONCLUSIONS

A methodology has been developed to provide confidence intervals on the probability of kill due to blast,  $P_{kb}$ , in the absence of fragmentation and aiming error. The assumption was made that the input parameters identifying  $P_{kb}$  were subject to error in such a way that such errors could be described as random variables defined by uniform distributions. Estimates for  $E[P_{kb}]$  and  $Var[P_{kb}]$  were provided for three different situations. These, in turn, were used to define the confidence intervals on  $P_{kb}$ .

**END**

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